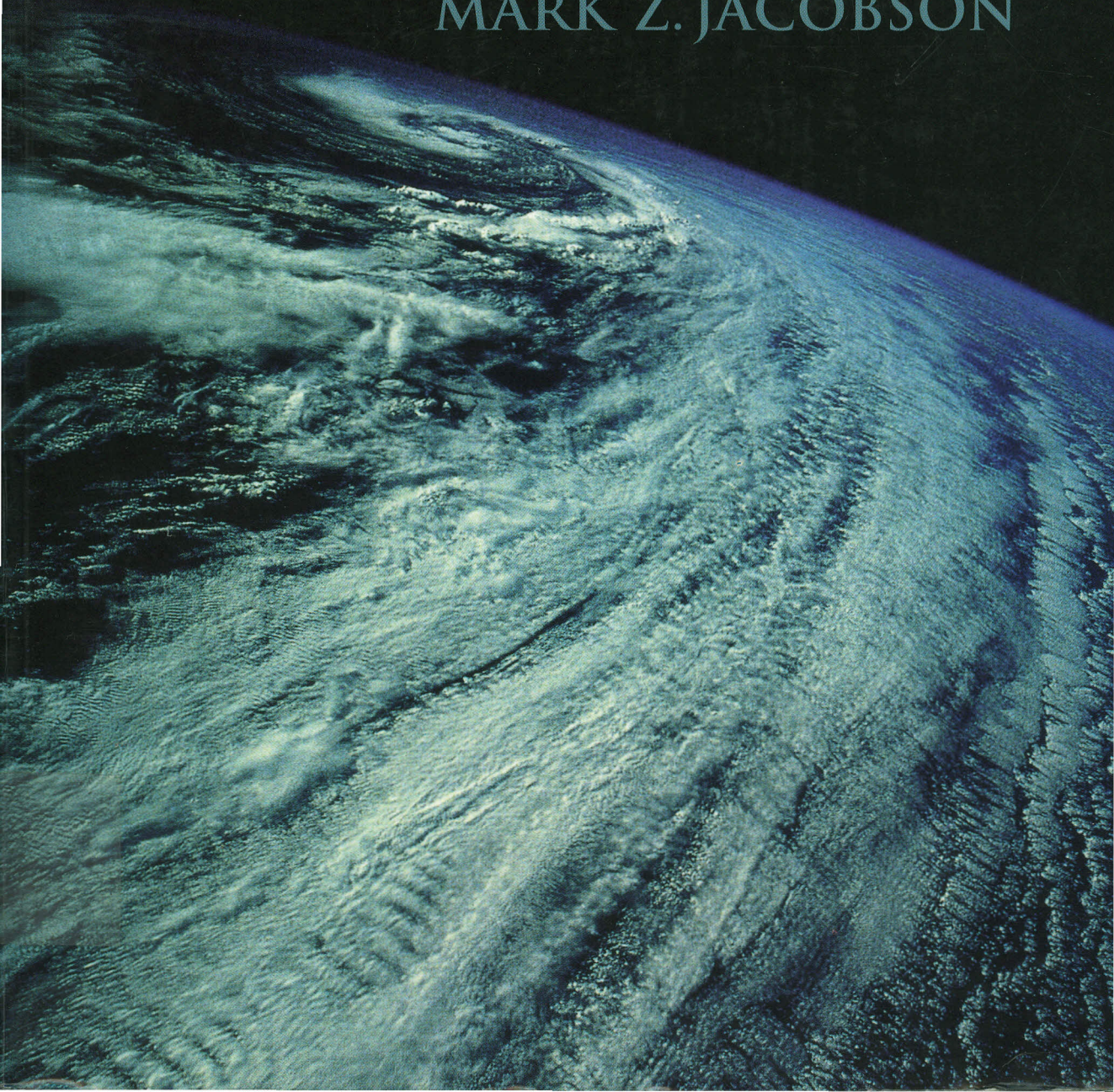


# FUNDAMENTALS OF ATMOSPHERIC MODELING

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# Fundamentals of Atmospheric Modeling

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